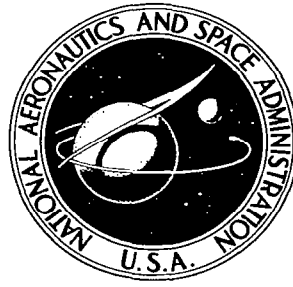


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**PROJECT "DIODE RELIABILITY
PREDICTION TECHNIQUE"**

by C. M. Ryerson

Prepared by

HUGHES AIRCRAFT COMPANY

Culver City, Calif.

for Goddard Space Flight Center



NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • FEBRUARY 1967



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By C. M. Ryerson

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Prepared under Contract No. NAS 5-9638 by
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SUMMARY

OBJECT OF PROJECT

This project has succeeded in its objective of developing a new approach to reliability prediction for semiconductor diodes based on realistic mathematical models. A new rationale for reliability modeling was developed by defining reasonable approximations and expressing in useable mathematical form the natural processes of degradation to failure under stress. A "law" of failure rate prediction was thus established for diodes. Some of the uses of this "law" can be summarized as follows:

1. To determine if a lot of parts is typical of the standard part.
2. To establish a new model for similar but different types.
3. To evaluate the differences between supposedly identical lots.
4. To compare products from different suppliers.
5. To evaluate consistency of Quality Control in a supplier's plant from lot to lot.
6. To compare the effectiveness of quality control between suppliers for the same type parts.
7. To establish new constants and models for different part types.
8. To purify and perfect the model to deeper levels of interaction simulation.

SCOPE OF WORK

The project was divided into two phases. Phase 1 was to formulate a basic mathematical model in the form of failure rate prediction equations for diodes taking into account those environmental and operational factors which exert an influence upon the basic failure rates. Phase 2 was to design an expedient test program to obtain statistically significant data for quantifying the coefficients and exponents in the basic model.

The scope of the development was to include both mechanical and electrical degradation parameters for all the common environmental and load stress factors plus any other important factors such as quality adjustment. Five basic types of silicon diodes were singled out for detailed study as is explained under scope and purpose in the introduction.

CONCLUSIONS BASED ON FINDINGS

A tractable model for expressing the principle natural processes of diode failure can be written and then validated by results from specified monostress and combined stress tests. This model is presented and explained herein. A design of experiment is also described for validating the coefficients and exponents. It is believed that this project has resulted in a major technical breakthrough in the area of reliability prediction.

Although the project emphasized specific application to certain types of silicon diodes the technical approach developed here should be useful with all other types of components including integrated microcircuits.

SUMMARY OF RECOMMENDATIONS

1. In regard to the specified silicon diodes:
 - a. The detailed monostress and combined stress tests should be performed to validate the constants in the model.
 - b. The resultant "law" for diode failure rate prediction should be publicized and made available for use by all NASA and other Government agencies.
2. In regard to other component parts:
 - a. The proven model should be expanded to allow for prediction of failure rates on other diode types.
 - b. The "law" should be modified and developed further to apply to other types of parts such as microcircuits.
3. In regard to the modeling techniques:
 - a. The engineering and statistical tools and techniques such as used on this and other related projects for modeling reliability should be developed into a handbook for general use. This Modeling Handbook would provide guidelines and techniques for generating practical mathematical models for the reliability of all types of component parts, and for making use of new and existing models in reliability predictions and evaluation. Detailed examples of the application of the new "law" of failure rate should be included.

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1. INTRODUCTION

1.1 SCOPE AND PURPOSE

The purpose of this project was to develop a new more fundamental approach to reliability prediction based on realistic mathematical models. The hope was to achieve a breakthrough from the theoretical side using a knowledge of the basic physics and mathematics to provide a bridge of rationale and technique for scientific guidance of reliability prediction. Heretofore the art of failure prediction has been based largely on empirical data and routine application of the Inverse Product Rule. Although much has been learned in recent years about the Physics of Failure in component parts, until this project was completed there has been no theoretical means for making use of this information in failure rate prediction. There has been no "law" of failure for prediction purposes. In effect this desired "law" has been developed on this project. The hope has been achieved.

In order to limit the scope of the project to a practical range the field of effort was centered on semiconductor diodes in general, on silicon diodes primarily and on five typical types for detailed analysis. These five basic types can be listed as follows:

1. General purpose
2. Computer and switching
3. Zener or reference
4. Power or rectifier
5. Varactor (variable reactance)

The range of parameters included in the program can be listed as follows:

1. Mechanical degradation (gross physical – macro structure)
2. Electrical degradation (chemical – micro structure)
3. Environmental stress factors
 - a. Shock (S)
 - b. Vibration (V)
 - c. Constant acceleration (A)
 - d. Temperature
 - Operating (T_o)
 - Junction (T_J)
 - e. Radiation (ionizing)
 - Particulate (U)
 - Nonparticulate (U_g)
4. Electrical load stress factors
 - a. Current
 - b. Power
 - c. Voltage
5. Quality adjustment factor

1.2 GENERAL BACKGROUND

1.2.1 Modeling Defined

The primary function of modeling is to express the processes of nature in mathematical form showing the operational relations between variables and parameters so that apparently diverse and obtusely related phenomena can be understood. A good model will reduce great quantities of experimental data to simple mathematical form without loss of meaning and with greatly increased visibility of important principles and interactions.

Unfortunately most processes of nature are so complicated that it is frequently impossible to develop mathematical formula which correspond exactly to the physical reality. Simplifying assumptions and approximation techniques must be used to make the model practical for engineering use. Over-simplification or over-approximation can also reduce the practical usefulness of the model. Thus for each set of model conditions there is an optimum compromise between simplicity of the equations, approximations used, engineering suitability and model effectiveness. In practice the equation is planned to include the most important features of the process with a minimum of assumptions so that the model will be fruitful for purposes of prediction and theoretical speculation. Second order effects and secondary features of the process must be ignored for complex processes.

1.2.2 Error and Proof of Models

In many cases the error introduced by simplifying assumptions and approximations used in deriving models can be evaluated in the same experiments used to evaluate the constants and exponents in the model. In other cases a separate series of experiments using modified models must be planned. From all this it becomes obvious that astute engineering judgment, coupled with a keen sense of the mathematical implications, must be employed both in modeling and in the design of experiments to validate the models.

The acid test for a model is empirical observation of the model results when it is exercised with specific test conditions. The simplest or purest model from the mathematical sense will be worthless to engineering unless it corresponds reasonably to physical observations. Occasionally several models may be found that account equally well for the observational data. In this case, and until the experimental results are sufficiently refined to favor one hypothesis over the others, the choice of model can be a matter of personal taste. Usually preference is given to the alternative hypothesis which is easiest to design into a validating experiment. Thus, apparent simplicity may not be the deciding factor in model selection.

1.2.3 Deliberate Levels of Complexity

Various complexities of models may be required to illustrate certain ranges or states of functional response. To illustrate this, consider the elementary concept of the "ideal gas law" in thermodynamics. This assumes that there is no attraction between the molecules and no interaction with the volume occupied. It can thus be stated that the product of the pressure and volume of one mole of a gas equals the product of the gas constant by the absolute temperature. This simple model can be expressed by the equation:

$$pv = RT \quad (1)$$

where

p = pressure

v = the specific volume

R = the gas constant

T = the absolute temperature

The assumption that the gas molecules are perfect elastic spheres is a considerable over-simplification, but this simple model provides an adequate description of the behavior of most gases over a wide range of pressure and temperature. When necessary this ideal gas law can

be modified by more realistic assumptions concerning the interactions of the molecules to yield a more accurate picture of the behavior of actual gases over any range of pressure and temperature. This becomes Van Der Waals Gas Equation:

$$\left(p + \frac{a}{v^2}\right) (v - b) = RT \quad (2)$$

where "a" and "b" are constants depending respectively on the cohesion between the molecules and the volume occupied by the molecules.

The first over-simplified equation is frequently a deliberate choice for use in specific cases where it has been found to apply with sufficient accuracy. For other purposes the more complex model is required. In other words, the application will determine the level of complexity or completeness required in the model.

This fact is important now at the state of the art in component part reliability control. In many cases, the use of a complex level model, true to every detail of interaction in the degradation and failure process, would be impractical to use even if it could be written. In fact, even after the "perfectly complete" model is developed for part reliability, many cases will exist where less complex models are more useful. With this fact in mind, this project has developed the highest order model which can be used now to cover a majority of diode prediction purposes. As the state of the art progresses, greater complexity can be added to account for a greater range of application and for second order effects.

1.3 PROJECT DIVISIONS

The work of the project was divided into two phases. The first phase was to develop background material and to formulate a basic mathematical model for predicting the failure rates of diodes from basic physical and environmental information. The various factors and part parameters which exert an influence on the basic failure rates were studied. A model was hypothesized and perfected over a several

month period to account for known theoretical and empirical phenomena. Each hypothetical theorem was tested in turn by exercising the developing model with typical empirical data.

The second phase of the project was to design an efficient experimental test program to quantify the coefficients and exponents in the basic model for the selected diode types.

Monthly status reports were submitted each month over an 8 month period.

1.4 WORK SUMMARY

1.4.1 General

The work of the project was divided into specific categories for monthly emphasis. The first two months were spent in laying the groundwork for subsequent effort. Plans were made, the literature on modeling and failure predicting was researched, and the major diode suppliers were queried for their possible input.

The third month's activity hinged about the investigation of the relation of temperature and electrical stress to diode failure rates. A major outcome of this month's activity was the conclusion that temperature is the prime interaction factor for each of the other stress factors. The basic form of the mathematical model was hypothesized on this basis in the third month. Later investigation confirmed this conclusion but added corrective terms to the model.

The fourth month's activity studied the relation between the environmental stresses of vibration, shock, and constant acceleration and the diode failure rate. Granting that the hypothesized model looked good for temperature and electrical stress this month's activity developed new terms to express the relations of these new stress factors.

The fifth month's activity studied the probable effect of radiation on the model. Much of this time was devoted to defining the nature of radiation to be considered and its known effect on diodes. The interaction of temperature and radiation was studied with the conclusion that this effect could be ignored in the model. No term was needed to relate temperature to radiation degradation in the normal operating temperature range. During this month the model was realigned to reflect the fact that the different stress factors affect different basic portions of the failure rate.

The sixth month's activity studied the effects of humidity and low pressure on the diode failure rate. The result of this effort was the conclusion that in a highest order model for diode failure rate no term is needed to provide for the effects of low pressure and humidity. In

an aside at this point, this conclusion would probably not apply to a highest order model for integrated circuits.

The seventh month's activity resulted in completion of the general equation (Mathematical Model) and the basic design of experiments to validate the constants.

The eighth and ninth months have been occupied with developing details of the experimental tests and preparing examples of the plan for inclusion in this final report.

These various work phases are illustrated in Figure 1.

CONTRACT NAS 5-9638 NASA, GODDARD	TASK 1						TASK 2					
	8/65 1	9/65 2	10/65 3	11/65 4	12/65 5	1/66 6	2/66 7	3/66 8	4/66 9	5/66 10	6/66 11	
PLANNING AND ANALYSIS												
IDENTIFICATION OF DIODE FAILURE MECHANISMS, MODES AND ENVIRONMENTS AFFECTING DIODE FAILURE RATE. CONSTRUCT TABLE RELATING THE ENVIRONMENTS AND FAILURE MECHANISMS AND MODES												
IDENTIFICATION OF RELATION BETWEEN TEMPERATURE AND ELECTRICAL STRESS ENVIRONMENTS AND DIODE FAILURE RATE												
IDENTIFICATION OF RELATION BETWEEN SHOCK, VIBRATION AND CONSTANT ACCELERATION ENVIRONMENTS AND DIODE FAILURE RATE												
IDENTIFICATION OF RELATION BETWEEN RADIATION ENVIRONMENTS AND DIODE FAILURE RATE												
IDENTIFICATION OF RELATION BETWEEN PRESSURE AND HUMIDITY ENVIRONMENTS AND DIODE FAILURE RATE												
PREPARATION OF EQUATIONS												
DESIGN OF EXPERIMENT												
TEST PLANNING AND DETAILING												
REPORTS												
MONTHLY PROGRESS		▲	▲	▲	▲	▲	▲	▲	▲			
FINANCIAL MANAGEMENT NASA FORM 533		▲	▲	▲	▲	▲	▲	▲	▲	▲		
FINAL REPORT DRAFT										▲		
FINAL REPORT APPROVED											▲	

Figure 1. Program plan, diode reliability prediction technique.

1.5 CONTENT OF REPORT

The following Technical Discussion section of this final report contains a brief explanation of the rationale for the model hypothesis, a description of the final model, the basic simplifying assumptions, the Design of Experiments, details of the quantification tests, instructions for applying the completed model and conclusions and recommendations resulting from the project. Following the Technical Discussion is a section containing Ancillary Material such as a bibliography, a glossary and various appendices.

2. TECHNICAL DISCUSSION

2.1 MODEL HYPOTHESIS RATIONALE

2.1.1 General Form of the Model

In answer to the Request for Proposal on this project Hughes proposed a model based on a modified form of the Eyring equation. It was explained that the final model might take the form:

$$\lambda_{U_i} = \lambda_{B_i} \pi_{E_{l_i}}^{E_{n_i}} \quad (3)$$

where

λ_{U_i} = Predicted failure rate of the i^{th} type or subtype under a given set of environmental and operational conditions.

λ_{B_i} = Base failure rate of the i^{th} type or subtype of diode under some defined set of laboratory-controlled, steady-state, "reference" conditions.

$\pi_{E_{l_i}}^{E_{n_i}}$ = The product of a series of Eyring factors from 1 to n relating the effect on the base failure rate of the i^{th} type of diode resulting from the interaction of the combined applied stresses.

It is interesting to note that when the final model was completed it closely resembled this original form. The final form for comparison is:

$$\lambda_{P_i} = \left[\lambda_a \pi \Phi (\text{Mech}) + \lambda_b \pi \Phi (\text{Elec}) \right] \cdot \Phi (Q) \quad (4)$$

where

λ_a = That portion of the base failure rate which is modified by the π factors related to macro physical and mechanical degradation, $\pi \Phi (\text{Mech})$.

λ_b = The portion of the base failure rate which is modified by the π factors related to electrical degradation based on internal micro and atomic mechanisms, $\pi\Phi$ (Elec).

and

$\Phi(Q)$ = A common modifying factor for supplier quality affecting both portions of the base failure rate.

The rationale and intermediate steps in the development of the model can be explained as in the following discussion.

2. 1. 2 The Eyring Model

The empirical success of modified forms of the Arrhenius equation to describe many sets of reliability data led to the suspicion that there must be a more comprehensive model which could be derived from fundamental physics and which would be based on other physical parameters in addition to the temperature and time rate of degradation as in the Arrhenius model. Several more general but Arrhenius type equations were proposed by Eyring (1936) for use in describing thermodynamic phenomena. The basic form of the Eyring model considers not only temperature and time rate of degradation but also Boltzmann's Constant, Planck's Constant, and the Activation Energy. All these quantities are involved in the basic physical response of component parts to environmental and loading stresses. The Eyring model thus seemed like a logical starting place to develop a reliability prediction model.

2. 1. 3 Absolute Rate Theory

The original starting point was taken from Absolute Rate Theory. The Eyring equation is expressed by T. L. Hill in "Introduction to Statistical Dynamics" (Addison-Wesley 1960) on Page 197 as:

$$K = \left(\frac{kT}{h} \right) \frac{(q^\ddagger / V) e^{-\Delta u_e^\ddagger / kT}}{(q^1 / V) (q^{1BC} / V)} \quad (5)$$

where

K is the rate constant and the q/V terms are physical constants for a given part type, thus;

$$K_p = C \left(\frac{kT}{h} \right) e^{\frac{-\Delta ue^\ddagger}{kT}} \quad (6)$$

where

Δue^\ddagger is the activation energy per molecule

but

$$R = k N_o \quad (7)$$

where

R = Universal gas constant

k = Boltzmann's Constant

N_o = Avagadro's Number

so if we multiply

$$\frac{\Delta ue^\ddagger}{kT} \times \frac{N_o}{N_o} = \frac{N_o \Delta ue^\ddagger}{RT} \quad (8)$$

then

$N_o \Delta ue^\ddagger$ is the activation energy per mole (1 gm molecular weight).

This converts (5) to:

$$K_p = C \left(\frac{kT}{h} \right) e^{-\frac{N_o \Delta ue^\ddagger}{RT}} \quad (9)$$

which becomes

$$c = C \frac{kT}{h} e^{-\frac{\Delta H - T \Delta S}{RT}} \quad (10)$$

where

ϵ = Reaction Rate (time rate of failure for a component)

k = Boltzmann's Constant

h = Planck's Constant

R = Universal Gas Constant

ΔH = Activation Energy in cal/mole

ΔS = Change in Entropy in cal/mole/°K

T = Temperature (Kelvin)

The transition from Equation (9) to (10) merely recognizes that some of the total input energy is absorbed by the material without causing degradation (entropy).

2. 1. 4 Hypothesized First Model

When Equation (10) is examined it is found to consist basically of:

$$\lambda_p = C \cdot f(\text{Strength}) \cdot f(\text{Stress}) \quad (11)$$

where λ_p = part failure rate

$f(\text{Strength})$ = a function of the strength inherent in the design of a particular type part to resist failure.

$f(\text{Stress})$ = a function of the stress energy conditions (environmental and loading) applied to the part.

Consequently, when the model was first hypothesized during the third month to provide for the effects of junction temperature, electrical stress loading, and a probable quality adjustment factor, the basic equation, after several false starts, finally became:

$$\lambda_{p1} = A Q \lambda_b \left[B e^{\left(\frac{T_A + \theta_A P_j + 273}{C} \right)^D} \right] \left[E e^{\left(\frac{S}{F} \right)^G} \right] \cdots \quad (12)$$

where

λ_{p1} = Failure rate for specific part

λ_b = Base failure rate for material and construction

Q = Adjustment for manufacturer's quality control
 A = Part type general adjustment factor
 B = Temperature interaction factor operator
 C = Knee of temperature mode No. 1 degradation
 D = Acceleration factor for temperature
 T_A = Ambient temperature (°C)
 θ_A = Thermal resistance junction to air (°C/W)
 P_j = Power dissipated by junction
 E = Electrical stress interaction factor operator
 F = Knee of stress mode No. 1 degradation
 G = Acceleration factor for stress
 S = Stress value

Tentative values were assigned to each of the constants to test the probable validity of this model. The results looked very good. Typical empirical data substituted into this model produced individual isothermal response curves similar to those in MIL-HDBK-217 and the RADC Notebook.

2. 1. 5 Adjusting the Activation Energy

It was not until later that it was recognized that this model was good only at a single temperature. To make it universal for any temperature, an additional interaction factor was needed which would, in effect, adjust the activation energy for the presence of the non-thermal stress at different temperatures.

To see this more clearly consider the basic form of Equation (12). This can be expressed as:

$$\lambda_p = C \cdot e^{\left(\frac{T}{n_T}\right)^D} \cdot e^{\left(\frac{S}{n_S}\right)^G} \quad (13)$$

where C, D and G are constants for a part type, n_T and n_S are knee values for the respective stress-strain response curves for temperature and another stress factor (S).

When this model was used in an attempt to develop a whole family of isostress curves, such as is found in MIL-HDBK-217, the family was distorted and did not faithfully resemble families derived from empirical data. The mathematical modeling was then "massaged" until a model was achieved which would faithfully reproduce whole families of curves in the form originally developed from empirical data.

2. 1. 6 Intermediate Model

The improved intermediate form of the model can be expressed as:

$$\lambda_{p1} = C \cdot Q \cdot \left[e^{\left(\frac{T}{n_T}\right)^D} \right] \cdot \left[e^{\left(\frac{S_1}{n_1}\right)^E} \left(\frac{T_o}{T_1}\right)^{EF} \right] \left[e^{\left(\frac{S_2}{n_2}\right)^G} \left(\frac{T_o}{T_1}\right)^{GH} \right] \cdot \left[e^{\left(\frac{S_3}{n_3}\right)^J} \left(\frac{T_o}{T_1}\right)^{JK} \right] \quad (14)$$

where

- C = A constant for a particular part design
- Q = A quality adjustment factor for a particular supplier
- n_i = The knee of specific stress response curves
- T = Temperature of Body (Kelvin)
- D = The Temperature Degradation Acceleration factor
- S_i = Non-thermal stress factor values
- E, G, J = Non-thermal stress acceleration factors
- F, H, K = Temperature – stress interaction factors
- $\frac{T_o}{T_1}$ = Ratio of operating temperature to normal derating temperature in degrees Kelvin.

2. 1. 7 The Effect of Failure Modes

The next major step forward in the evolution of the final model came about in connection with the study into the effect on the model

of the various failure modes. Early in this study it became obvious that there are usually two predominant failure modes existing in most diodes. One of these relates to mechanical macro-structure failures and the other to electrical micro-structure failures. The predominantly mechanical stress factors modify the portion of the base failure rate relating to the mechanical or macro mode and the electrical and nuclear (radiation) stress factors modify the portion of the base failure rate relating to the micro mode. This improvement in the model can be understood better by considering the following discussion.

The general equation was originally hypothesized to be of the form:

$$\lambda_{p'} = \lambda_a + \lambda_b \cdot \pi \Phi (\text{Mod}) \quad (15)$$

where

$\lambda_{p'}$ = Failure rate for a specific part.

λ_a = Base failure rate for a particular part design which is constant for the part type and unaffected by the variables in manufacture and use.

λ_b = That portion of the base failure rate which varies according to the conditions of manufacture and application.

$\pi \Phi (\text{Mod})$ = Factors which modify the variable portion of failure rate related to both electrical and mechanical degradation mechanisms.

The study on the implications of different failure modes led to the revision of the model to the final general form:

$$\lambda_{p'} = \left[\lambda_a \pi \Phi (\text{Mech}) + \lambda_b \pi \Phi (\text{Elec}) \right] \cdot \Phi (Q) \quad (16)$$

where

λ_a = That portion of the base failure rate which is modified by the π factors related to macro physical and mechanical degradation, $\Phi (\text{Mech})$.

λ_b = The portion of the base failure rate which is modified by the π factors related to electrical degradation based on internal micro and atomic mechanisms, $\pi \Phi$ (Elec).

and

$\Phi(Q)$ = A common modifying factor for supplier quality affecting both portions of the base failure rate.

2. 1. 8 Standard Nomenclature

A standard nomenclature was also established so that the form of each π factor can be expressed as:

$$\Phi(\alpha, T_o) = e^{\left\{ \left[\frac{\alpha}{n} \left(\frac{T_o}{T_1} \right)^I \right]^C + K \right\}} \quad (17)$$

where

α = The stress parameter.

n = A value representing the knee of the degradation curve for the primary mode affected by the particular stress (α).

T_o = The operating temperature (degrees Kelvin).

T_1 = The reference derating temperature (usually 298°K).

I = The temperature to α interaction factor.

C = Acceleration factor for the stress α

K = Part type general adjustment factor

This leads directly to the final model which is described in the next section.

*

2.2 THE MATHEMATICAL MODEL

The final mathematical model is shown in two forms. The first is the model itself which reveals the primary relation between the cause and effect factors. This model is then modified by combining the modifying K constants into a single term which is easier to prove in the design of experiments. These two forms are shown here as Equations (18) and (19) where:

λ_a = That portion of the base failure rate which is modified by the π factors related to macro physical and mechanical degradation, $\pi \Phi(\text{Mech})$.

λ_b = The portion of the base failure rate which is modified by the π factors related to electrical degradation based on internal micro and atomic mechanisms, $\pi \Phi(\text{Elec})$.

S = Shock

V = Vibration

A = Acceleration

T_o = Operating temperature

$E = \frac{E_o}{E_m}$ = Electrical loading ratio

E_o = Operating load

E_m = Maximum rating electrical load

T_J = Junction temperature = $T_a + \theta_a P_J + 273$

U = Radiation rate (particle type)

U_G = Radiation rate (non-particle type)

Q = Quality factor

n = A value representing the knee of the degradation curve for the primary mode affected by the particular stress.

The Model – Equation 18

$$\begin{aligned}
 \lambda_P^1 = \lambda_a \cdot e & \left\{ \left[\frac{S}{n_S} \left(\frac{T_o + 273}{298} \right)^{I_S} \right]^{C_S} + K_S \right\} \cdot e \left\{ \left[\frac{V}{n_V} \left(\frac{T_o + 273}{298} \right)^{I_V} \right]^{C_V} + K_V \right\} \cdot \\
 & e \left\{ \left[\frac{A}{n_A} \left(\frac{T_o + 273}{298} \right)^{I_A} \right]^{C_A} + K_A \right\} \cdot e \left\{ \left[\frac{T_o + 273}{n_T} \right]^{C_T} + K_T \right\} \cdot e \ln Q \\
 & + \lambda_b \cdot e \left\{ \left(\frac{T_o + \theta_a P_J + 273}{n_T} \right)^{C_T} + K_T \right\} \cdot e \left\{ \left[\frac{E_o}{n_E E_n} \left(\frac{T_o + 273}{298} \right)^{I_E} \right]^{C_E} + K_E \right\} \cdot \\
 & e \left\{ \left[\frac{U k_U}{n_U} \right]^{C_U} + K_U \right\} \cdot e \left\{ \left[\frac{U_G k_G}{n_G} \right]^{C_G} + K_G \right\} \cdot e (\ln Q)
 \end{aligned}
 \tag{18}$$

General Model

Modified Basic Model

$$\begin{aligned}
 \lambda_{P1} = \lambda_a \cdot e & \left(\frac{S}{n_S} \right)^{C_S} \left(\frac{T_o + 273}{298} \right)^{I_S C_S} \left(\frac{V}{n_V} \right)^{C_V} \left(\frac{T_o + 273}{298} \right)^{I_V C_V} \\
 & \left(\frac{A}{n_A} \right)^{C_A} \left(\frac{T_o + 273}{298} \right)^{I_A C_A} \left(\frac{T_o + 273}{n_T} \right)^{C_T} \cdot e^{(\ln Q)} \cdot e^{K_a} + \\
 & \lambda_b \cdot e \left(\frac{T_o + \theta_a P_J + 273}{n_T} \right)^{C_T} \left(\frac{E_o}{n_E E_m} \right)^{C_E} \left(\frac{T_o + 273}{298} \right)^{C_E I_E} \left(\frac{U k_U}{n_U} \right)^{C_U} \\
 & \left(\frac{U_G k_G}{n_G} \right)^{C_G} \cdot e^{(\ln Q)} \cdot e^{K_b}
 \end{aligned} \tag{19}$$

where

$$e^{K_a} = e^{(K_s + K_V + K_A + K_T)} \tag{20}$$

and

$$e^{K_b} = e^{(K_T + K_E + K_U + K_G)} \tag{21}$$

This modified general equation (19) is thus the final form of the model containing the constants to be evaluated in a designed experiment.

2.3 SIMPLIFYING ASSUMPTIONS

As was pointed out in the introduction during the explanation of Reliability Modeling the successful and realistic model must be based on an optimum combination of simplifying assumptions for a practical approximate fit to the processes of nature pictured in the model.

A major contribution of this project has been to define the necessary assumptions required to achieve a practical model for the failure rate of diodes. These can be summarized as in the following list.

1. There is only one predominant electrical failure mode and one mechanical mode to be considered in a practical mathematical model for silicon diodes. All other modes are of secondary importance and can be ignored in the model.
2. Synergistic effects between the basic modes are of second order importance and can be ignored in the model.
3. The primary interaction effect is between temperature and all the other stress factors. Each term of the model must include provision for temperature-stress interaction. All other stress-stress interactions can be ignored in the model.
4. The quality control in each supplier's plant, including process and raw material control, can be assumed to affect each of the failure modes by a like amount. Thus a single quality adjustment factor can be used for each supplier in all portions of the model for each part type.
5. The model need consider only catastrophic type failure modes or the equivalent degradation modes which occur suddenly.
6. The failure rate of parts, when normalized to quality λ_p / Q , equals the sum of (the basic residual failure rate due to the primary mechanical mode as modified by the mechanical and macro effecting stresses) plus (the basic residual failure rate due to the primary electrical mode as modified by the electrical, chemical, and molecular affecting stresses), i. e.,

$$\frac{\lambda_p}{\Phi Q} = \lambda_a \pi \Phi (S_{\text{Mech}}) + \lambda_b \pi \Phi (S_{\text{Elec}}) \quad (22)$$

7. The storage failure rate for conditions of no stress other than real time is numerically equal to the sum of λ_a and λ_b modified by a quality factor; i. e. ,

$$\lambda_p (\text{Storage}) = (\lambda_a + \lambda_b) \Phi Q \quad (23)$$

8. The failure rate for parts is related exponentially to the failure activation energy plus the entrophy or the energy absorbed in the component materials which does not contribute to the degradation of the part.
9. The value (n_T) representing the knee of the temperature degradation curve is the same for both the mechanical failure mode and the electrical failure mode.

2.4 DESIGN OF EXPERIMENT

Because of the many parameters and constants inherent in a true mathematical representation of all the physical processes involved in diode failure rate prediction, even the simplified model developed on this project would be too complex for solution by a direct statistical attack in a full factorial experiment. Fortunately, it was possible to design the model to consist of independent π terms which can be evaluated independently.

The approximate probable values for each of the constants were hypothesized as the terms in the basic equation were established so that it was possible later to design a very specific series of experiments to obtain the exact values of the constants with a minimum amount of testing. This was possible based on a high degree of confidence, from other factors, that the model is sound and tractable.

In a very real sense, this project has all been oriented toward developing a practical scientific experiment. To a practical degree, the philosophy of Bayesian Statistics has been employed all during the model hypothesis. In other words, all previous known information on the subject, both theoretical and empirical, was considered for its direct impact and fringe implications at each step of the development. Then, as the model was formulated, each version was tested by the insertion of typical constants from actual data so that the results of model exercise could be compared with actuality. The final step of experiment design was a simple detailing of tests to obtain the actual values of constants for specific diode types.

The final designed experiment to validate the constants and exponents can be summarized briefly in three steps as follows:

1. Select the best supplier for each general type of part (e. g. , General Purpose Diode from Texas Instruments or Fairchild, etc.). Include all five types, General Purpose, Computers and Switching, Zener, Power, and Varactor.
2. Perform monostress tests by varying one stress factor at a time and evaluate the constants, exponents, and basic portions of the failure rate λ_a and λ_b of the general equation.
3. Perform a series of combined environment tests for several suppliers. Evaluate the results of these tests to determine the value of the Q term for each supplier.

A more detailed procedure for this design can be summarized in the following statements:

2.4.1 Mono-Stress Experiments

2.4.1.1 Select one type of diode at a time from the best supplier for evaluation (e.g., General Purpose Diode from Texas Instrument or Fairchild). By selecting the best supplier, $Q = 1$ and $\ln Q = 0$. By this action the term $e^{\ln Q} = 1$.

2.4.1.2 Hold all other stress factors at zero level while each stress factor and its interaction with temperature is evaluated in turn.

2.4.1.3 Select three levels of the stress factor under investigation to be evaluated at three temperatures. The levels of stress chosen are to be in 1/2:1:2 ratio so that the test results will define a "relative" isothermal response curve for the stress at each test temperature. This experiment can be illustrated to this point as a two factor experiment involving nine data cells. This is illustrated in Figure 2.

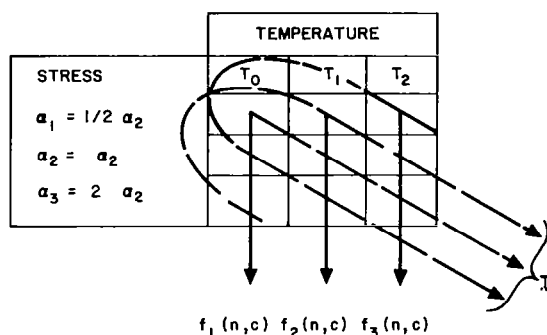


Figure 2. Two factor experiment (to derive values of n , C , and I).

2.4.1.4 From the isothermal data obtained at $T_0 = 25^\circ\text{C}$, which is at normal room ambient conditions, we can evaluate the "knee" values of (n) and acceleration factor (C) values.

2.4.1.5 From several sets of isothermal data, we can evaluate the values of the interaction (I) factor.

2.4.1.6 Repeat the experiment steps 2.4.1.2 through 2.4.1.5 for each stress factor in turn. This will establish the values for all the constants in the equation except the Q factor and the λ 's.

2.4.1.7 In order to evaluate the K factors it will first be necessary to establish the relative weighted values of λ_a and λ_b for each of the diode types under consideration.

In other words, the relative probability must be determined for the mechanical or electrical failure mode occurring for a given diode design and construction. Past experience can be used to make this determination. This has been done for use in the design of experiment and the results can be illustrated as shown in Table 1.

Here it is shown that there is a 1 to 2 ratio between λ_a and λ_b for Point Contact diodes. That is, there is twice the likelihood that a mechanical failure will occur than an electrical failure. For the alloy type there is three times the likelihood that an electrical failure will occur. Finally for the Diffused Mesa or Planar types the odds are 2 to 1 in favor of the electrical failure.

Type of Construction	Ratio of λ_a to λ_b
Point Contact	$\lambda_a = 2 \lambda_b$
Alloy	$\lambda_a = 1/3 \lambda_b$
Diffused Mesa or Planar	$\lambda_a = 1/2 \lambda_b$

Table 1. Ratio of λ_a to λ_b for a given diode design and construction.

where

λ_a = Portion of base failure rate related to macro physical and mechanical degradation which is modified by the π factors.

λ_b = Portion of the base failure rate related to electrical degradation based on internal micro and atomic mechanisms which is modified by the π factors.

2.4.1.8 The relative ratios determined in step 2.4.1.7 are used to determine the K factors (K_a and K_b in Equation (19)) by the use of selected values in the General Equation (19). These selected values are as follows:

1. All stress levels set at zero.
2. Reference temperature $T_o = 25^\circ\text{C}$.
3. Select best vendor so that $\ln Q = 0$.

By this action the term $e^{\ln Q} = 1$ and all other terms become 1 except for the temperature and K terms. Thus the Equation (19) reduces to:

$$\lambda_P^1 = e^{\left(\frac{298}{n_T}\right)^{C_T}} \left[\lambda_a e^{K_a} + \lambda_b e^{K_b} \right] \quad (24)$$

4. To evaluate $\lambda_a e^{K_a}$ and $\lambda_b e^{K_b}$, solve Equation (19) for several values of one stress factor while the other stress factors are held at a zero level and $T_o = 25^\circ\text{C}$. For example, select the shock term for this analysis. Then Equation (19) becomes:

$$\lambda_P^1 = e^{\left(\frac{S}{n_S}\right)^{C_S}} \cdot \lambda_a e^{K_a} e^{\left(\frac{298}{n_T}\right)^{C_T}} + \lambda_b e^{K_b} e^{\left(\frac{298}{n_T}\right)^{C_T}} \quad (25)$$

All the constants in this equation have previously been determined. By selecting three values of S such that the effect of the electrical failure term ($\lambda_b \theta(T)$) is negligible the Equation (25) reduces to:

$$\lambda_P^1 = e^{\left(\frac{S}{n_S}\right)^{C_S}} \cdot \lambda_a e^{K_a} e^{\left(\frac{298}{n_T}\right)^{C_T}} \quad (26)$$

5. Solving Equation (26) for known data from step 2.4.1.2 provides the value of $\lambda_a e^{K_a}$. A similar operation with emphasis on the electrical terms will provide the value of $\lambda_b e^{K_b}$.
6. Using the relationships in Table 1, based on the type of diode under consideration, solve Equation (24) for the values of K_a and K_b using specific values of λ_p as measured in step 2.4.1.2.

2.4.2 Combination Stress Experiments

A combined environment experiment for each type of diode will be performed after the constants of the general equation have been determined by evaluating one stress factor at a time for that part type. Four suppliers will be evaluated for each type of diode. For each supplier, a combined environment experiment will be performed using the 16 test combinations shown in Table 2. All 64 test combinations of the four suppliers are to be tested in random order.

The plan represents a full factorial and the data can be analyzed by an analysis of variance. There is no cell replication and it will be necessary to use the highest order interaction as an estimate of error.

This combined environment experiment is necessary to prove the accuracy of the constants, exponents and interaction factors and to prove the validity of the model for predicting failure rates for combinations of the stress factors. Any variations of results between suppliers for the same part type is a measure of the quality ratings for the suppliers. It was thought originally that it would be necessary to perform a series of vendor surveys to evaluate this Q term of the general equation for each supplier. Subsequent results during the design of experiments reveal that it will probably be better to measure the Q value for each supplier as a part of a combined environment experiment. This Q rating combines design, material, process, fabrication, and quality control capability of each supplier.

Supplier I

Test Combination	Factor Levels			
	V Vibration	E Voltage	UG Radiation	T Temperature
1	V ₁	E ₁	U ₁	T ₁
2	V ₁	E ₁	U ₁	T ₁
3	V ₁	E ₁	U ₂	T ₁
4	V ₁	E ₁	U ₂	T ₂
5	V ₁	E ₂	U ₁	T ₁
6	V ₁	E ₂	U ₁	T ₂
7	V ₁	E ₂	U ₂	T ₁
8	V ₁	E ₂	U ₂	T ₂
9	V ₂	E ₁	U ₁	T ₁
10	V ₂	E ₁	U ₁	T ₂
11	V ₂	E ₁	U ₂	T ₁
12	V ₂	E ₁	U ₂	T ₂
13	V ₂	E ₂	U ₁	T ₁
14	V ₂	E ₂	U ₁	T ₂
15	V ₂	E ₂	U ₂	T ₁
16	V ₂	E ₂	U ₂	T ₂
Stress levels: V ₁ , E ₁ , U ₁ = 50 percent of maximum rating V ₂ , E ₂ , U ₂ = Maximum rating T ₁ = 25°C T ₂ = 125°C				

Table 2. Combined environment test matrix.

2.5 TEST DETAILS

As previously described the evaluation experiment will involve two types of testing. Both Mono-Stress tests and Combination Stress tests will be included. These are explained in some detail in the following discussion.

2.5.1 Mono-Stress Tests

The procedure for the test program to evaluate the constants and exponents of the general equation can be summarized in the following statements:

2.5.1.1 Choose one type of diode to be evaluated and select best supplier. For example: General Purpose diode, 1N 485B from Texas Instruments.

2.5.1.2 Hold all other stress factors at zero level while each stress factor and its interaction with temperature is evaluated in turn. Each test cell in the experiment matrix will require 10 units as the sample size to be tested. The figure of merit entered into the data cell will be the best estimate of failure rate λ which will occur when 50 percent of the sample fails (i.e., five out of 10). If f is the number of failed units and T is the total observed test time (total of times-to-failure of failed units, plus operating times of non-failed units) then the best estimate of $\hat{\lambda}$, the failure rate and \hat{m} , the part mean-time-to failure are:

$$\hat{\lambda} = \frac{f}{T} \quad (V-1)$$

$$\hat{m} = \frac{T}{f} \quad (V-2)$$

For example: Ten diodes begin an electrical load test at accelerated test conditions. Five failures are observed at 500, 2000, 3500, 4000 and 5000 hours, respectively, when the test is terminated after the fifth failure.

$$T = 500(1) + 2000(1) + 3500(1) + 4000(1) + 5000(6) = 40,000 \text{ hours}$$

$$f = 5$$

$$\hat{\lambda} = \frac{5}{40,000} = 0.000125 \text{ failure per hour}$$

$$\hat{m} = \frac{40,000}{5} = 8000 \text{ hours}$$

Record value of $\lambda = 0.000125$ in the appropriate data cell as the best estimate of failure rate.

2.5.1.3 Perform a Physics of Failure Analysis of each failed diode to identify the failure mode and mechanism that caused them to fail under the conditions in each Test Matrix data cell. The cause of failure for each of the 5 failed diodes in the same data cell will be reviewed in order to eliminate any spurious failure data. For example, if 10 diodes are subjected to an electrical load test at accelerated test conditions at 25°C. Test is terminated after 5 failures are observed. Physics of Failure Analysis is performed on each failed part and causes of failure determined to be the following:

- a. First failure – Open due to broken lead
- b. Second failure – Degradation due to junction imperfection
- c. Third failure – Degradation due to junction imperfection
- d. Fourth failure – Degradation due to junction imperfection
- e. Fifth failure – Degradation due to junction imperfection

Review of the causes of failure might lead to the possibility that the cause of the first failure, i.e., a broken lead, might be faulty or spurious failure data. Hence the experiments might be justified in discarding this observation and treating the data as if the spurious or faulty failure data did not exist.

2.5.1.4 To evaluate the values of the temperature constants (n_T) and (C_T), perform the experiment under conditions specified in Test Matrix I.

- a. λ_{p_1} is "shelf-life" data and sometimes can be obtainable from the vendor. When this is possible it should correlate with the information from cell 1 of Test Matrix I.
- b. λ_{p_2} and λ_{p_3} will be used to find values of (C_T) and (n_T) of the general equation.

2.5.1.5 To evaluate the values of (n_S) , (C_S) , and (I_S) of the general equation, perform the experiment specified by Test Matrix II. Data obtained may also be used to evaluate value of the constant $\lambda_a e^{Ka}$.

2.5.1.6 To evaluate the values of (n_V) , (C_V) , and (I_V) of the general equation, perform the experiment specified by Test Matrix III. Data obtained may also be used to evaluate value of the constant $\lambda_a e^{Ka}$.

2.5.1.7 To evaluate the values of (n_A) , (C_A) , and (I_A) of the general equation, perform the experiment by Test Matrix IV. Data obtained may also be used to evaluate value of the constant $\lambda_a e^{Ka}$.

2.5.1.8 To evaluate the values of (n_E) , (C_E) , and (I_E) of the general equation, perform the experiment specified by Test Matrix V. Data obtained may also be used to evaluate value of the constant $\lambda_b e^{Kb}$.

2.5.1.9 To evaluate the values of (n_U) and (C_U) of the general equation, perform the experiment specified by Test Matrix VI.

2.5.1.10 To evaluate the values of (n_{UG}) and (C_{UG}) of the general equation, perform the experiment specified by Test Matrix VII.

TEST MATRIX I

Stress	Temperature °C		
	T ₀	T ₁	T ₂
Set all stress factors at zero level except temperature S = 0 V = 0 A = 0 E = 0 U = 0 U _G = 0	λ'_{P_1}	λ'_{P_2}	λ'_{P_3}

TEST MATRIX II

Stress – Set all stress factors at zero level except shock and temperature	Temperature °C		
	T ₀ 25	T ₁ 85	T ₂ 125
S ₁ = 1000 g (0.5 millisecond)	λP_{S_1}	λP_{S_4}	λP_{S_7}
S ₂ = 2000 g (0.5 millisecond)	λP_{S_2}	λP_{S_5}	λP_{S_8}
S ₃ = 4000 g (0.5 millisecond)	λP_{S_3}	λP_{S_6}	λP_{S_9}

TEST MATRIX III

Stress – Set all stress factors at zero level except vibration and temperature	Temperature °C		
	T ₀ 25	T ₁ 85	T ₂ 125
V ₁ = 30 g	λP_{V_1}	λP_{V_4}	λP_{V_7}
V ₂ = 60 g	λP_{V_2}	λP_{V_5}	λP_{V_8}
V ₃ = 120 g	λP_{V_3}	λP_{V_6}	λP_{V_9}

TEST MATRIX IV

Stress – Set all stress factors at zero level except acceleration	Temperature °C		
	T ₀ 25	T ₁ 85	T ₂ 125
A ₁ = 20,000 g	λP_{A_1}	λP_{A_4}	λP_{A_7}
A ₂ = 40,000 g	λP_{A_2}	λP_{A_5}	λP_{A_8}
A ₃ = 80,000 g	λP_{A_3}	λP_{A_6}	λP_{A_9}

TEST MATRIX V

Stress – Set all stress factors at zero level except voltage E_i where $E_i = EO/EM$ EO = operating EM = rated	Temperature °C		
	T ₀ 25	T ₁ 85	T ₂ 125
$E_1 = 0.5$	λP_{E_1}	λP_{E_4}	λP_{E_7}
$E_2 = 1.0$	λP_{E_2}	λP_{E_5}	λP_{E_8}
$E_3 = 2.0$	λP_{E_3}	λP_{E_6}	λP_{E_9}

TEST MATRIX VI

Stress – Set all factors at zero level except radiation rate (particle type)	Temperature °C 25
$U_1 = 5 \times 10^9$ neutrons/cm ²	λP_{U_1}
$U_2 = 1 \times 10^{10}$ neutrons/cm ²	λP_{U_2}
$U_3 = 2 \times 10^{10}$ neutrons/cm ²	λP_{U_3}

TEST MATRIX VII

Stress – Set all factors at zero level except radiation rate (non-particle type)	Temperature °C 25
$U_{G_1} = 5 \times 10^7$ roentgens	$\lambda P_{U_{G_1}}$
$U_{G_2} = 1 \times 10^8$ roentgens	$\lambda P_{U_{G_2}}$
$U_{G_3} = 2 \times 10^8$ roentgens	$\lambda P_{U_{G_3}}$

2.5.1.11 Evaluation of Temperature Constants. To evaluate the value of the temperature constants (n_T) and (C_T), perform the experiment shown in Table 3.

Stress	Temperature °C		
	T_0 25	T_1 85	T_2 125
Set all stress factors at zero level			
$S = 0$			
$V = 0$			
$A = 0$			
$E = 0$	λ_{P_1}	λ_{P_2}	λ_{P_3}
$U = 0$			
$U_G = 0$			

Table 3. Experiment to evaluate temperature constants.

From Table 3 the following data results:

- a. Test cell 1 is a "shelf-life" data and sometimes can be obtained from the supplier.
- b. Test cells 2 and 3 data will be used to evaluate C_T and n_T .
- c. By selecting the best vendor, $Q = 1$ and $\ln Q = 0$. By this action the term $e^{\ln Q} = 1$ and all other terms in Equation (19) become 1 except for the temperature, the K terms and the 'S. Thus when $T_0 = 25^\circ\text{C}$, the Equation (19) reduces to:

$$\lambda_{P_1} = e^{(298/n_T) C_T} \left[\lambda_a e^{K_a} + \lambda_b e^{K_b} \right] \quad (27)$$

- d. When $T_0 = 85^\circ\text{C}$, Equation (19) reduces to:

$$\lambda_{P_2} = e^{(358/n_T) C_T} \left[\lambda_a e^{K_a} + \lambda_b e^{K_b} \right] \quad (28)$$

- e. When $T_0 = 125^\circ\text{C}$, Equation (19) reduces to:

$$\lambda_{P_3} = e^{(398/n_T) C_T} \left[\lambda_a e^{K_a} + \lambda_b e^{K_b} \right] \quad (29)$$

Rearranging Equations (28) and (29), we can solve for the values of C_T and n_T as follows:

$$\left[\lambda_a e^{K_a} + \lambda_b e^{K_b} \right] = \frac{\lambda_{P_2}}{e^{(358/n_T) C_T}} \quad (30)$$

$$\left[\lambda_a e^{Ka} + \lambda_b e^{Kb} \right] = \frac{\lambda_1}{p_3} \frac{C_T}{e^{(398/n_T)}} \quad (31)$$

$$\ln \frac{\lambda_1}{p_3} = (398/n_T) C_T \quad (32a)$$

and

$$\ln \frac{\lambda_1}{p_2} = (358/n_T) C_T \quad (32b)$$

$$\frac{\ln \frac{\lambda_1}{p_3}}{\ln \frac{\lambda_1}{p_2}} = \frac{(398/n_T) C_T}{(358/n_T) C_T} = \left[\frac{(398/n_T)}{(358/n_T)} \right]^{C_T} = (398/358)^{C_T} = (1.1117)^{C_T}$$

$$\ln \frac{\ln \frac{\lambda_1}{p_3}}{\ln \frac{\lambda_1}{p_2}} = C_T \ln 1.1117$$

$$C_T = \frac{\ln \left(\frac{\ln \frac{\lambda_1}{p_3}}{\ln \frac{\lambda_1}{p_2}} \right)}{\ln 1.1117} \quad (33)$$

from Equations (32a) and 32b),

$$\ln \ln \frac{\lambda_1}{p_3} = C_T \ln (398/n_T)$$

and

$$\ln \ln \lambda_{\frac{1}{p_2}} = C_T \ln (358/n_T)$$

$$\frac{\ln \ln \lambda_{\frac{1}{p_3}}}{\ln \ln \lambda_{\frac{1}{p_2}}} = \frac{C_T \ln (398/n_T)}{C_T \ln (358/n_T)}$$

Let

$$Z = \frac{\ln \ln \lambda_{\frac{1}{p_3}}}{\ln \ln \lambda_{\frac{1}{p_2}}}$$

$$Z = \frac{\ln 398 - \ln n_T}{\ln 358 - \ln n_T}$$

$$Z \ln 358 - Z \ln n_T = \ln 398 - \ln n_T$$

$$Z \ln n_T - \ln n_T = Z \ln 358 - \ln 398$$

$$\ln n_T (Z - 1) = Z \ln 358 - \ln 398$$

$$\ln n_T = \frac{Z \ln 358 - \ln 398}{Z - 1}$$

$$n_T = \text{antilog} \left[\frac{\frac{\ln \ln \lambda_{\frac{1}{p_3}}}{\ln \ln \lambda_{\frac{1}{p_2}}} \ln 358 - \ln 398}{\frac{\ln \ln \lambda_{\frac{1}{p_3}}}{\ln \ln \lambda_{\frac{1}{p_2}}} - 1} \right]$$

(34)

2.5.1.12 Evaluation of exponents and "knee" values of the other stress factors of the general equation.

To evaluate the values of the constants n_S , C_S and I_S , perform the experiment shown in Table 4.

Stress – Set all stress factors to zero level except shock listed below	Temperature, degrees C		
	T ₀ 25	T ₁ 85	T ₂ 125
$S_1 = 1/2 S_2$	① λP_{S_1}	④ λP_{S_4}	⑦ λP_{S_7}
$S_2 = S_2$	② λP_{S_2}	⑤ λP_{S_5}	⑧ λP_{S_8}
$S_3 = 2 S_2$	③ λP_{S_3}	⑥ λP_{S_6}	⑨ λP_{S_9}

Table 4. Two factor experiment.
(To derive values for n, C, and I)

- Set all stress factors at zero level except shock and temperature.
- Select three levels of shock in 1/2:1:2 ratio.
- By selecting best supplier the term $e^{\ln Q} = 1$ and Equation (19) becomes:

$$\lambda P_S = e^{(S/n_S)^{C_S}} \lambda_a e^{K_a} e^{(T_0+273/n_T)^{C_T}} + \lambda_b e^{K_b} e^{(T_0+273/n_T)^{C_T}} \quad (35)$$

The values of n_T and C_T have previously been determined from Equations (33) and (34).

By selecting values of shock term S such that the effect of the electrical term is negligible the Equation (35) reduces to

$$\lambda P_S = e^{(S/n_S)^{C_S}} \lambda_a e^{K_a} e^{(T_0+273/n_T)^{C_T}} \quad (36)$$

We can use Equation (36) and data obtained from test cells 1, 2, and 3 of Table 4 to find the value of n_S , C_S and the constant $\lambda_a e^{K_a}$.

When the test conditions of Table 4 are performed, data obtained in test cells 1, 2, and 3 are used in Equation (36) as follows:

$$\lambda P_{S_1} = e^{(S_1/n_S)^{C_S}} \lambda_a e^{K_a} e^{(298/n_T)^{C_T}}$$

Let

$$\psi = \lambda_a e^{K_a} e^{(298/n_T)^{C_T}}$$

$$\lambda P_{S_1} = \psi e^{(S_1/n_S)^{C_S}} \quad (37)$$

$$\lambda P_{S_2} = \psi e^{(S_2/n_S)^{C_S}} \quad (38)$$

$$\lambda P_{S_3} = \psi e^{(S_3/n_S)^{C_S}} \quad (39)$$

$$\ln \lambda P_{S_1} = \ln \psi + \left(\frac{\frac{1}{2} S_2}{n_S} \right)^{C_S} \quad (40)$$

where

$$S_1 = \frac{1}{2} S_2$$

$$\ln \lambda P_{S_2} = \ln \psi + \left(\frac{S_2}{n_S} \right)^{C_S} \quad (41)$$

where

$$S_2 = S_2$$

$$\ln \lambda P_{S_3} = \ln \psi + \left(\frac{2 S_2}{n_S} \right)^{C_S} \quad (42)$$

where

$$S_3 = 2 S_2$$

$$\ln \lambda P_{S_1} = \ln \psi + \left(2^{-1} \right)^{C_S} \left(\frac{S_2}{n_S} \right)^{C_S} \quad (43)$$

$$\ln \lambda P_{S_2} = \ln \psi + \left(2^0 \right)^{C_S} \left(\frac{S_2}{n_S} \right)^{C_S} \quad (44)$$

$$\ln \lambda P_{S_3} = \ln \psi + \left(2^{+1} \right)^{C_S} \left(\frac{S_2}{n_S} \right)^{C_S} \quad (45)$$

Let

$$Z_i = \ln \lambda P_{S_i}$$

$$Y = \ln \psi$$

$$X = \left(\frac{S_2}{n_S} \right)^{C_S}$$

Therefore, Equations (43), (44), and (45) become:

$$Z_1 = Y + X Z^{-C_S} = Y + X \left(Z^{C_S} \right)^{-1} \quad (46)$$

$$Z_2 = Y + X = Y + X \left(Z^{C_S} \right)^0 \quad (47)$$

$$Z_3 = Y + X Z^{C_S} = Y + X \left(Z^{C_S} \right)^{+1} \quad (48)$$

Let

$$\theta = Z^{C_S}$$

$$Z_1 = Y + X \theta^{-1} = Z_2 - X + X \theta^{-1} \quad (49)$$

$$Z_2 = Y + X \theta^0 = Y + X \quad (50)$$

$$Z_3 = Y + X \theta^{+1} = Z_2 - X + D \theta \quad (51)$$

$$\theta = \frac{X}{Z_1 - Z_2 + X} = \frac{Z_3 - Z_2 + X}{X} \quad (52)$$

$$Z_1 Z_3 - Z_1 Z_2 + X Z_1 - Z_2 Z_3 + Z_2^2 - X Z_2 + X Z_3 - X Z_2 + X^2 = X^2$$

$$Z_1 Z_3 - Z_2 Z_3 - Z_1 Z_2 + Z_2^2 + X (Z_1 - Z_2 + Z_3 - Z_2) = 0$$

$$(Z_1 - Z_2) (Z_3 - Z_2) + X [(Z_1 - Z_2) + (Z_3 - Z_2)] = 0$$

$$X = \frac{-(Z_1 - Z_2) (Z_3 - Z_2)}{(Z_1 - Z_2) + (Z_3 - Z_2)} \quad (53)$$

using value of X from Equation (53) one can find θ and C_S

$$\theta = \frac{Z_3 - Z_2 + X}{X}$$

$$C_S = \frac{\ln \theta}{\ln 2} \quad (54)$$

knowing X, S_2 , and C_S we can find n_S

$$X = \left(\frac{S_2}{n_S} \right)^{C_S} \quad \text{and} \quad X^{1/C_S} = \frac{S_2}{n_S}$$

Therefore

$$n_S = \frac{S_2}{X^{1/C_S}} \quad (55)$$

We can then evaluate the value of the constant $A = \lambda_a e^{K_a}$ from Equation (38):

$$\lambda P_{S_2} = \psi e^{(S_2/n_S)^{C_S}}$$

where

$$\psi = \lambda_a e^{K_a} e^{(T_0 + 273/n_T)^{C_T}}$$

$$\psi = \frac{\lambda P_{S_2}}{e^{(S_2/n_S)^{C_S}}} \quad (56)$$

After finding the value of ψ and having previously found values of n_T and C_T , we can evaluate the value of the constant $A = \lambda_a e^{K_a}$ from:

$$\lambda_a e^{K_a} = \frac{\psi}{e^{(T_0 + 273/n_T)^{C_T}}} \quad (57)$$

After the values of the "knee" (n_S) and constants (C_S) and $\lambda_a e^{K_a}$ have been determined, we can find the value of exponent (I_S) by using the data in test cells 7, 8, and 9 which are obtained under the following conditions:

- Set all stress factors to zero level except shock term S and temperature.
- Set temperature T_0 = maximum rated temperature of 125°C.
- By selecting best supplier the term $e^{\ln Q} \approx 1$.
- Select values of shock term S such that the effect of the electrical term is negligible and by this action Equation (19) reduces to:

$$\lambda P_S = \lambda_a e^{K_a} e^{(125+273/n_T)^{C_T}} e^{(S/n_S)^{C_S}} \left(\frac{125+273}{298} \right)^{C_S I_S} \quad (58)$$

Since we have previously found the values of n_T , C_T , n_S , C_S , and $\lambda_a e^{K_a}$, we can solve for the value of the only unknown factor I_S .

$$I_S = \frac{\ln \left(\frac{\lambda P_{S8}}{\lambda_a e^{K_a} e^{(398/n_T)^{C_T}}} \right)}{\ln \left(\frac{(S_2/n_S)^{C_S}}{(398/298)^{C_S}} \right)} \quad (59)$$

We can then proceed to evaluate the "knee" value (n_1), acceleration factor (C_1), interaction factor (I_1), constants $\lambda_a e^{K_a}$ and $\lambda_b e^{K_b}$ of another stress factor at a time by the same method as above.

See Appendix A for a specific example of a solution to the general equation using assumed values of failure rates.

2.5.1.13 Mechanical Degradation Test Environments. Shock, vibration, and acceleration environments are expressed in terms of gravity units g. The Stress Factors S, V, or A, will involve peak amplitude, duration of stress environment application, and repetition rate or duty cycle. Failure rate for relays and switches is expressed in failures per million operations and can be converted to failures per million hours by multiplying by the repetition rate per hour. (Failure rate/million operations \times operations/hour = failure rate/million hours.) This method may be adapted to convert the mechanical environmental test failure data from shock, vibration, and acceleration into failure rate per million hours or failure rate in percent per 1000 hours, whichever units are desired.

The following methods are recommended to establish the procedure necessary to determine the test conditions of each mechanical degradation environmental test.

a. Shock. Shock will be expressed in gravity units g per specified time duration per blow or impact. Select the critical plane if known, it not use evaluation plan Appendix B. After the critical plane of orientation most likely to induce failure due to shock has been determined, the experiment as specified in Test Matrix II shall be performed in that plane with 10 diodes for each test cell in the experiment matrix. The diodes shall be nonoperating and shall be subjected to the specified g levels, time duration and temperature conditions. One test cycle shall consist of five impacts or blows. The diodes shall be visually examined and measurement of electrical characteristics will be made prior to start of testing. The diodes shall be examined for evidence of catastrophic failure such as open or short-circuiting after each test cycle. The test shall continue until five out of 10 diodes in each test cell in the experiment matrix have failed. The best estimate of failure rate shall be recorded in the appropriate test cell of the experiment matrix. From the data obtained, we can evaluate the values of (n_S), (C_S), and (I_S).

b. Vibration. Before the experiment Test Matrix III is performed, it will be necessary to determine the critical plane of orientation most likely to induce failure due to vibration. If this is not known, perform the evaluation plan as described in Appendix C.

After the critical plane of orientation most likely to induce failure due to vibration has been determined, the experiment specified in Test Matrix III shall be performed in that plane with 10 diodes for each test cell. The diodes shall be nonoperating and shall be subjected to a simple harmonic motion having an amplitude varied to maintain peak acceleration of specified g value. One test cycle shall consist of a frequency sweep between the approximate limits of 10 and 2000 cps which shall be traversed in 25 ± 5 minutes. When resonance is detected, the diodes shall be vibrated for 5 minutes at each critical resonant frequency observed. Interruptions are permitted provided the requirements for maintaining a constant peak acceleration of specified g value, rate of change and test duration are met. The testing shall continue until five out of 10 diodes in each test cell have failed. The best estimate of the failure rate shall be recorded in the appropriate test cell of the experiment matrix. From the test data obtained from experiment Test Matrix III, we can evaluate the "knee" value (n_V) and exponents (C_V) and (I_V).

c. Acceleration. Before the experiment Test Matrix IV is performed, it will be necessary to determine the critical plane of orientation most likely to induce failure due to acceleration. If this is not known, perform the evaluation plan as described in Appendix D.

After the critical plane of orientation most likely to induce failure due to acceleration has been determined, the experiment specified in Test Matrix IV shall be performed in that plane with 10 diodes for each test cell.

The diodes shall be nonoperating and shall be subjected to the specified g value. The rate of acceleration shall be increased smoothly from zero to the specified g value in not less than 20 seconds. The rate of deceleration shall be decreased smoothly to zero in not less than 20

seconds. One test cycle shall consist of subjecting the diodes for 1 minute at the specified g level. The diodes shall be examined visually and measurements of electrical characteristics will be made prior to start of testing. The diodes shall be examined for evidence of catastrophic failure such as open or short-circuiting after each test cycle. The testing shall continue until five out of 10 diodes in each test cell have failed. The best estimate of the failure rate shall be recorded in the appropriate test cell block of the experiment matrix. The data obtained will be used to evaluate the "knee" value (n_A) and the exponents (C_A) and (I_A).

2.5.1.14 Electrical Degradation Test Environments

a. Electrical Loading Test. Perform the experiment as specified in Test Matrix V with 10 diodes for each test cell. The diodes shall be examined visually and measurements of electrical characteristics will be made prior to start of testing. The diodes shall be subjected to the conditions specified in Test Matrix V. The testing shall continue until five out of 10 diodes in each test cell shall failed. The best estimate of the failure rate data obtained shall be recorded in the appropriate test cell in the experiment matrix. The data obtained will be used to evaluate the values of (n_E), (C_E), and (I_E).

b. Radiation Rate (Particle Type). Perform the experiment as specified in Test Matrix VI with 10 diodes for each test cell. The diodes shall be examined visually and measurements of electrical characteristics will be made prior to start of testing. The diodes shall be subjected to conditions specified in Test Matrix VI. The testing shall continue until five out of 10 diodes in each test cell have failed. The best estimate of the failure rate data obtained shall be recorded in the appropriate test cell of the experiment matrix. The data obtained will be used to evaluate the values of (n_U) and (C_U).

c. Radiation Rate (Non-Particle Type). Perform the experiment as specified in Test Matrix VII with 10 diodes for each test cell. The diodes shall be examined visually and measurements of electrical

characteristics will be made prior to start of testing. The diodes shall be subjected to conditions specified in Test Matrix VII. The testing shall continue until five out of 10 diodes in each test cell have failed. The best estimate of the failure rate data obtained shall be recorded in the appropriate test cell of the experiment matrix. The data obtained will be used to evaluate the values of (n_{UG}) and (C_{UG}) .

2.5.2 Combination Stress Tests

There is a practical upper limit to the number of environmental factors such as temperature, vibration, electrical loading, shock, acceleration, radiation, etc. that may be examined in an experimental program. To establish a practical combined environment test, it is proposed to combine vibration, electrical loading, radiation (non-particle type, gamma rays) and temperature as shown in Table 2. The following combined environment experiment is setup on this basis. However, if facilities for combining other environments can be made available by NASA Goddard, Hughes will be happy to develop an appropriate alternate combined environment test plan.

Four suppliers will be evaluated for each type of diode. For each supplier, a combined environment experiment will be preformed using the test combinations shown in Table 2, with 10 diodes in each test cell. All 64 test combinations of the four suppliers are to be tested in random order.

The plan represents a full factorial and the data can be analyzed by an analysis of variance. In a completely balanced experiment, each level of each factor is tested at all levels of all the other factors so that the total number of experiments required is the product of all the levels of all the factors. Each of four environmental stresses, as shown in Table 2, will be tested at two levels and the fifth factor (supplier) at four levels. Therefore, the combined environment stress test program would be $2 \times 2 \times 2 \times 2 \times 4$ or 64 separate measurements.

Each of the samples from each supplier will be exposed to a different combination of combined stress, as shown in Table 2, for long periods of time. The data entered in Table 5 shall be actual failure

Suppliers	V ₁								V ₂							
	E ₁				E ₂				E ₁				E ₂			
	U _{G1}		U _{G2}		U _{G1}		U _{G2}		U _{G1}		U _{G2}		U _{G1}		U _{G2}	
	T ₁	T ₂	T ₁	T ₂	T ₁	T ₂	T ₁	T ₂	T ₁	T ₂	T ₁	T ₂	T ₁	T ₂	T ₁	T ₂
S ₁																
S ₂																
S ₃																
S ₄																
Total																

Table 5. Failure rate data -- combination stress tests.

rate results. For example, if test diodes from Supplier S₁ were tested in accordance with the different combinations of the combined stresses shown in Table 2, the test results would be recorded as shown in Table 6. Repeat the combined stress experiments with diodes from three other suppliers (S₂, S₃, and S₄) with the test combinations shown in Table 2 in random order and record the test results in Table 5.

The following procedure outlines a general method for an analysis of variation evaluation of test results of the Combined-Stress experiment.

1. Tabulate the failure-rate data from the Combined-Stress experiment in Table 5.
2. Write the symbols for the main factors and the combinations of the factors and tabulate in Table 7.
3. Calculate x_1 , x_2 and $(\sum x)^2/N, \sum x$ will be the sum of all the values in the bottom of Table 5 and $N = 64$.

Suppliers	V ₁								V ₂							
	E ₁				E ₂				E ₁				E ₂			
	U _{G1}		U _{G2}		U _{G1}		U _{G2}		U _{G1}		U _{G2}		U _{G1}		U _{G2}	
	T ₁	T ₂	T ₁	T ₂	T ₁	T ₂	T ₁	T ₂	T ₁	T ₂	T ₁	T ₂	T ₁	T ₂	T ₁	T ₂
S ₁	λ1	λ2	λ3	λ4	λ5	λ6	λ7	λ8	λ9	λ10	λ11	λ12	λ13	λ14	λ15	λ16
S ₂																
S ₃																
S ₄																
Total																

Table 6. Failure rate data — combination stress tests from experiment in accordance with Table 2.

4. Evaluate all the main-factor effects. For example: vibration.
 - a. Obtain the sum of squares of that main-factor total divided by the number of measurements in each total, less correction term.
 - b. For example, vibration totals level $V_1 = \sum \lambda v_1$

$$\text{level } V_2 = \sum \lambda v_2$$

$$\text{Vibration sum of squares } V = \frac{(\sum \lambda v_1)^2 + (\sum \lambda v_2)^2}{32} - \frac{(\sum x)^2}{64}$$
 - c. Record this value in Table 7 in the sum of squares column for source V.
 - d. Evaluate the other main-factor effects and record results in Table 7.
5. Evaluate all first-order interactions for the different pairs of factors. Table 8 shows an example for the totals of a pair of factors such as Vibration (V) and Electrical Loading (E). The number subscripts indicate the level of the factor involved.

Source	Sum of Squares	Degrees of Freedom	Mean Squares
V (Vibration)		1	
E (Elec. Stress)		1	
U (Radiation)		1	
T (Temperature)		1	
S (Supplier)		3	
VE		1	
VU		1	
VT		1	
VS		3	
EU		1	
ET		1	
ES		3	
UT		1	
US		3	
TS		3	
VEU		1	
VET		1	
VES		3	
VUT		1	
VUS		3	
VTS		3	
EUT		1	
EUS		3	
ETS		3	
UTS		3	
VEUT		1	
VEUS		3	
VETS		3	
VUTS		3	
EUTS		3	
VEUTS		3	
Total		63	

Table 7. Analysis of variance, combination stress tests.

	V_1	V_2
E_1	$\Sigma\lambda_{V_1E_1}$	$\Sigma\lambda_{V_2E_1}$
E_2	$\Sigma\lambda_{V_1E_2}$	$\Sigma\lambda_{V_2E_2}$

Table 8. First-order interactions.

- a. Obtain the totals for the different pairs of factors and tabulate as in Table 8.
 - b. Calculate the sum of squares for these different pairs, such as VE, VT, etc.
 - c. Record results in Table 7 in the sum of squares column for the different pairs of factors.
6. Evaluate the second-order interactions for the different combinations of three factors. Table 9 shows an example of a combination of three factors such as Vibration (V), Electrical Loading (E), and Temperature (T). The number subscripts indicate the level of the factor involved.

	V_1		V_2	
	E_1	E_2	E_1	E_2
T_1	$\Sigma\lambda_{V_1E_1T_1}$	$\Sigma\lambda_{V_1E_2T_1}$	$\Sigma\lambda_{V_2E_1T_1}$	$\Sigma\lambda_{V_2E_2T_1}$
T_2	$\Sigma\lambda_{V_1E_1T_2}$	$\Sigma\lambda_{V_1E_2T_2}$	$\Sigma\lambda_{V_2E_1T_2}$	$\Sigma\lambda_{V_2E_2T_2}$

Table 9. Second-order interactions.

- a. Obtain the totals for the different combinations of three factors and tabulate as in Table 9.
 - b. Calculate the sum of squares for these different combinations of three factors such as VEU, VET, etc.
 - c. Record results in Table 7 in the sum of squares column for the different combinations of factors.
7. In the same manner as step 6, evaluate the third-order and fourth-order interactions and record the results in Table 7.
 8. Calculate the mean squares for each main-factor, and the combinations of the factors by obtaining the sum of squares and dividing by the number of degrees of freedom of that factor. Record the results in the mean squares column of Table 7.
 9. Without replication, the highest order interaction is usually taken as the estimate of error variance and is used for testing the significance of the other mean squares. Any variations of results between suppliers should correlate with the quality survey ratings obtained on the suppliers to validate the equation Q term.
 10. After completing the Combined Environment Tests and verifying the variation of results for the same type of diode is due to differences between suppliers, we can evaluate the value of the Q term for each supplier since this is the only unknown term. For the Combined Environment Tests, Equation (19) becomes:

$$\begin{aligned}
 \lambda_{p'} = & \lambda_a e^{K_a} e^{\ln Q} e^{\left(\frac{T_0 + 273}{n_T}\right)^{C_T}} e^{\left(\frac{V}{n_V}\right)^{C_V}} e^{\left(\frac{T_0 + 273}{298}\right)^{C_V I_V}} \\
 & + \lambda_b e^{K_b} e^{\ln Q} e^{\left(\frac{T_0 + \theta_a P_J + 273}{n_T}\right)^{C_T}} \\
 & e^{\left(\frac{E_0}{n_E E_m}\right)^{C_E}} e^{\left(\frac{T_0 + 273}{298}\right)^{C_E I_E}} e^{\left(\frac{U_G}{n_G}\right)^{C_G}}
 \end{aligned} \tag{60}$$

or,

$$\lambda_{p_1}' = e^{\ln Q} \left[\lambda_a e^{K_a} e^{\left(\frac{T_0+273}{n_T}\right)^{C_T}} e^{\left(\frac{V}{n_V}\right)^{C_V}} \left(\frac{T_0+273}{298}\right)^{C_V I_V} + \lambda_b e^{K_b} e^{\left(\frac{T_0+\theta_a P_J+273}{n_T}\right)^{C_T}} e^{\left(\frac{E_0}{n_E E_m}\right)^{C_E}} \left(\frac{T_0+273}{298}\right)^{C_E I_E} e^{\left(\frac{U_G}{n_G}\right)^{C_G}} \right] \quad (61)$$

The values of all the exponents and constants of Equation (61) except the Q term have previously been determined from the Mono-Stress Tests, hence the only unknown term is Q. By setting the Q term of the best supplier equal to 1, we can solve for the value of the Q term for each of the other suppliers by solving Equation (61) for that particular supplier using known data. For example for the same type of diode:

Let

λ_{p_1}' = failure rate obtained for best supplier

λ_{p_2}' = failure rate obtained for Supplier 2

Q_1 = 1 (quality term for best supplier)

Q_2 = quality term of Supplier 2 to be determined

Since we have previously determined the values of all the constants and exponents of Equation (61) except the Q term from the Mono-Stress Tests, we can set all the other terms on the right side of Equation (61), except Q term, equal to a constant C. Therefore, our results for best supplier and supplier S_2 becomes:

$$\lambda_{p_1}' = e^{\ln Q_1} [C] \quad (62)$$

$$\lambda_{P_2}' = e^{\ln Q_2} [C] \quad (63)$$

$$[C] = \frac{\lambda_{P_1}'}{e^{\ln Q_1}} = \frac{\lambda_{P_2}'}{e^{\ln Q_2}} \quad (64)$$

$$e^{\ln Q_2} = \frac{\lambda_{P_2}'}{\lambda_{P_1}'} e^{\ln Q_1} \quad (65)$$

By selecting best supplier $Q_1 = 1$ and $e^{\ln Q_1} = 1$, therefore

$$e^{\ln Q_2} = \frac{\lambda_{P_2}'}{\lambda_{P_1}'} \quad (66)$$

We can evaluate the value of Q_2 for Supplier 2 from Equation (66) which is the only unknown term; we can evaluate the values of the Q terms of the other suppliers in the same manner using the relationship expressed in Equation (66).

2.5.3 Test Part Summary

1. Mono-Stress Tests

Quantities of Parts Required					
Test Matrix	General Purpose	Computer	Zener or Reference	Power or Rectifier	Varactor
I	20	20	20	20	20
II	90	90	90	90	90
III	90	90	90	90	90
IV	90	90	90	90	90
V	90	90	90	90	90
VI	30	30	30	30	30
VII	30	30	30	30	30
Totals	440	440	440	440	440

2. Combined Environment Tests

Quantities of Parts Required				
Type of Diode	Supplier			
	1	2	3	4
General Purpose	160	160	160	160
Computer or Switching	160	160	160	160
Zener or Reference	160	160	160	160
Power or Rectifier	160	160	160	160
Varactor	160	160	160	160

3. Summary of Mono-Stress and Combined Environment Tests

Quantities of Parts Required				
Type of Diode	Supplier			
	1	2	3	4
General Purpose	600	160	160	160
Computer of Switching	600	160	160	160
Zener or Reference	600	160	160	160
Power or Rectifier	600	160	160	160
Varactor	600	160	160	160

4. Tests to Determine Critical Plane (If Required)

Quantities of Parts Required			
Type of Diode	Environment		
	Shock	Vibration	Acceleration
General Purpose	15	15	15
Computer of Switching	15	15	15
Zener or Reference	15	15	15
Power or Rectifier	15	15	15
Varactor	15	15	15

2. 6 APPLICATION INSTRUCTIONS

The preceding sections present the new "Law" for diode failure rate and describe a detailed experiment for validating the constants which apply to specific diodes. This section discusses some uses of the validated model.

Once the "law" of failure rate for a specific diode type is established as it will be in this proposed experiment for five different types, this information can be used in a variety of ways. These uses can be described in connection with specific conditions serving as constraints for the model application in given situations. Some of these situations and the use of the "law" can be summarized as follows:

1. To determine if a lot of parts is typical of the standard part.
2. To establish a new model for similar but different types.
3. To evaluate the differences between supposedly identical lots.
4. To compare products from different suppliers.
5. To evaluate consistency of Quality Control in a supplier's plant from lot to lot.
6. To compare the effectiveness of quality control between suppliers for the same type parts.
7. To establish new constants and models for different part types.
8. To purify and perfect the model to deeper levels of interaction simulation.

In all of these applications the "law" is used as a stepping stone and starting place as well as a frame of reference for comparing test data results. From the "law" and a knowledge of the Physics of Failure it is easy to select one or two major test parameters which will define accurately the critical response of the lots in question. The test results on two or three data points for these indicator parameters will quickly reveal differences between test lots and between these and the standard type.

For example, two or three data points in a mono-stress test can determine with high confidence that a given test sample is typical or different from the standard. A few more data points from crucial mono-stress tests plus a few data points from a combined environment test can generate an entirely new model.

2.7 CONCLUSIONS AND RECOMMENDATIONS

It is concluded that a major break-through has been achieved in the field of reliability theory and prediction. This project has succeeded in its objective of developing a new approach to the prediction of failure rates for semi-conductor diodes based on a realistic mathematical model. A new rationale has been developed for reliability modeling by defining practical simplifying assumptions and approximation techniques.

A tractable mathematical model for expressing the principle natural processes of diode failure has been formulated and a designed experiment has been developed for validating the constants. Illustrative examples of typical test results and probable values of constants are shown. This material can now be used for the actual validation test program.

Although the project emphasized specific types of silicon diodes the technical approach developed and the simplifying techniques should be useful with all other types of components including integrated microcircuits.

With this background in mind the following recommendations are made a part of this report.

1. In regard to the specified types of silicon diodes:
 - a. The detailed mono-stress and combined stress tests should be performed to validate the constants in the model
 - b. The resultant "Law" for diode failure rate prediction should be publicized and made available for use by all NASA and other Government agencies.
2. In regard to other component parts:
 - a. The proven model should be expanded to allow for prediction of failure rates on other diode types.
 - b. The "Law" should be modified and developed further to apply to other types of parts such as microcircuits.

3. In regard to the modeling techniques:

- a. The engineering and statistical tools and techniques such as used on this and other related projects for modeling reliability should be developed into a handbook for general use. This Modeling Handbook would provide guidelines and techniques for generating practical mathematical models for the reliability of all types of component parts, and for making use of new and existing models in reliability predictions and evaluations. Detailed examples of application of the new "Law" of failure rate should be included.

See Appendix E for an informal but detailed proposal for the preferred contents of this handbook.

3. ANCILLARY MATERIAL

The following section contains material on new technology, bibliography, glossary, and appendices with supplemental detailed information.

3.1 NEW TECHNOLOGY

The following innovations are being reported pursuant to the New Technology clause of this contract. Since there is no hardware associated with this project the developments do not represent inventions in the usual sense. However, important new technology has been developed in the following technique areas:

1. The formulation of a realistic and tractable mathematical model for expressing the "law" of diode failure rate.
2. The definition of practical simplifying assumptions and approximation techniques needed to formulate the above model as described herein.

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3.3 GLOSSARY OF TERMS

Acceleration - The rate of change in velocity either in speed or direction. It is usually expressed in "g" or gravity units.

Analysis of Variance - Separation of total sum-of-squares variations (from the mean) into components which can be assigned to variations between the classes or subclasses according to which the data are classified. These constituent portions of the sums of squares indicate, through mean-squares, the magnitude of class differences. The extent to which they differ from the residual mean square is a test of the hypothesis that such differences are governing the situation.

Ambient - Refers to the conditions of the surrounding medium, for example, temperature, pressure, etc., are ambient conditions.

Ambient Temperature - Temperature of the surrounding medium, such as gas or liquid, which comes in contact with the device.

Arithmetic Mean - The sum of a set of values divided by the number in the set.

Assignable Cause - A factor contributing to the variation in quality that is economically feasible to identify. Assignable causes must be identified and removed to attain statistical control.

Average – A value which represents or summarizes the relevant features of a set of values. (Commonly used to refer to arithmetic mean.)

Catastrophic Failure – A sudden change in the operating characteristics of some part or parameter resulting in complete failure of the item. For example: open or short circuits, structural failure, etc.

Chance Failure – A failure which occurs at random within the operational time of an equipment after all efforts have been made to eliminate design defects, unsound components, and before wear-out becomes predominant. (See also Random Failure.)

Characteristic – A trait, property or quality of a specified item, type of item, or groups of items.

Classification of Defects – Enumeration of possible defects of the device or unit of product classified according to their importance.

Component and Part Reliability – A component or part is reliable when it will operate to a predetermined level of probability under its maximum electrical ratings at the most severe combination of environments for which it was designed, for the number of hours and duty cycle of the end equipment to which it is applied, without failures exceeding the rate tolerable to the satisfactory functioning of the end equipment.

Controlled Process – A process which yields samples where characteristics remain consistently within the control limits of a predetermined Control Chart.

Correlation – A relationship between two occurrences which is expressed as a number between minus one and plus one.

Defect – Any deviation of a device from specified requirements. A device may contain more than one defect.

Degradation – Gradual deterioration in performance.

Degradation Factor – The factor that mathematically relates the part degradation rate to the associated stress.

Degradation Failure – A degradation failure is a failure that results from a gradual change in performance characteristics of an equipment or part with time. Failure due to the gradual deterioration of a characteristic or property of the device.

Deviation – The variation from the trend. In statistics, the difference between a particular number and the mean or average of the set of numbers under consideration.

Deviation, Standard Deviation– The amount of dispersion or scatter of the values from some mean value. The square root of the arithmetic mean of the squares of the deviations from the mean.

Displacement – The distance which a device under test is moved. In vibration, this usually is the distance of travel from one excursion extreme to the other.

Empirical – Relying on or proceeding on the information to be derived from experience and observation for lack of other knowledge; proceeding strictly experimentally by the trial and error method.

Environment – The aggregate of all the conditions and influences which affect the operation of equipment and components, e.g., physical location and operating characteristics of surrounding equipments and/or components; temperatures, humidity and contaminants of surrounding air, acceleration, shock and vibration, radiation; operational procedures; method of utilization, etc.

Environmental Tolerance – The ability of a system or portion thereof to operate within a specified environmental range. Examples: operation within a great range of temperature, input voltage, vibration, etc.

Factor – Something that contributes to the production of a result. Any of the numbers, elements, quantities, or symbols in mathematics that when multiplied together form a product.

Failure – Is a detected cessation of ability to perform a specified function or functions within previously established limits on the area of interest. It is a malfunction which is beyond adjustment by the operator

by means of controls normally accessible to him during the routine operation of the device. This requires that measurable limits be established to define satisfactory performance of the function.

Failure Mechanism – Refers to the manner in which the failure took place and subsequently became evident in the form of a particular failure mode. Failure mechanisms include the physical and chemical processes and the related chain of cause and effect events which cause the device to fail.

Failure Mode – Failure mode is used in this report to describe the final effect or failed state of the device (diode). For example, open or short-circuit.

Failure Rate – Is the frequency of failure occurrence over a period of time.

"g" or gravity unit – Measure or value of the earth's gravitational pull, or of a force required to accelerate or decelerate any freely movable body at the rate of approximately 32.16 feet per second.

Hypothesis – A proposition tentatively assumed in order to draw out its logical or empirical consequences and so to test its accord with facts that are known or may be determined.

Interaction – Mutual or reciprocal action or influence. For example, whenever two or more bodies exert mutual forces on one another, they are said to interact.

In statistics, when x is a function of y . And if the variation in x associated with given changes in y is affected by the values assumed by a third variable z , there is interaction between y and z . Thus, in the analysis of variance, if rows and columns represent different values of y and z , and the x values are the values in the cells, no interaction exists if the variation among the values from column to column is the same for each row. Interaction exists if the pattern of variation is different among the rows. For example, in the following 2 by 2 table, there is interaction, since the pattern of variation in values in Row 1 is different from the pattern in Row 2.

3	6
2	1

Law of Failure Rate Prediction – Is the validated mathematical model developed by this report and modified for application to specific part types.

Mathematical Model – Is a method used to express the processes of Nature in mathematical form showing the operational relationship between variables and parameters so that apparently diverse and obtusely related phenomena can be understood.

Macro-Physical – Relating to macro-physics.

Macro-Physics – Part of physics that deals with bodies large enough to be directly and individually observed and measured.

Macro-Structure – The structure of a material (as of metal) revealed by visual examination with little or no magnification.

Micro-Physical – Relating to micro-physics.

Micro-Physics – Physics of molecules, atoms and elementary particles.

Micro-Structure – The structure of a material (as an alloy or other crystalline mass) on a minute scale as revealed by the microscope or other means.

Mean – The sum of a set of values divided by the number in the set. See Arithmetic Mean and Average.

Parameter – A quantity of value which remains constant within a given set of conditions – i. e., is subject to change only if the conditions change.

Part (Detail Part, Component Part) – An individual electrical circuit element which, is an independent body, cannot be further reduced or divided without destroying its function. Examples: resistor, capacitor, diode, transistor, relay, connector, etc.

Part Failure Rate – That rate at which a part fails to perform its intended function.

Particulate – Existing in the form of minute separate particles, of or relating to distinct particles.

Phenomena – Any observable fact or event. In scientific usage, any fact or event of scientific interest susceptible of scientific description and explanation.

Population – The total collection of units from a common source; the conceptual total collection of units from a process, such as a production process. Also used in the sense of a "universe (or population) of observation." NOTE: universe, population and parent distribution are synonymous terms.

Statistical methods are based on the concept of a distribution of an exceedingly large number of observations, termed an infinite universe or population. An individual observation, the x of a sample, etc., may be thought of as one coming from a parent distribution or infinite population of like items. Statistical quality control is based on distributions in the population domain as contrasted to the time domain for reliability.

Prediction – An inference regarding a future event based on probability theory.

Probability – The limiting relative frequency in an infinite random series. If an event can occur in n ways and fail in m ways, and if these $m + n$ ways are equally likely, then the mathematical probability that the event will occur in any one trial is the ratio $n/(m+n)$.

In other words, the probability of an event is the theoretical relative frequency with which it will occur, such relative frequency being the ratio of the number of times the event is observed under experimental conditions to the total of a great number of observations made under those conditions.

Quality – The quality of a device is a measure of the degree to which it conforms to specification, design and workmanship standards. Its numerical rating is obtained by measuring the percentage defective of a lot or population at a given time.

Quality Characteristics – Those properties of an item or process in the population domain which can be measured, reviewed or observed, and which are identified in the drawings, specifications and contractual requirements.

Random – Unpredictable, without any definite rule, method, direction or order.

Random Failure – Any catastrophic failure whose probability of occurrence is variant with time, and whose occurrence within any given interval of time is, consequently, unpredictable.

Randomness – An equal chance for any of the possible outcomes.

Random Sample – A random sample is one in which each item in the lot has an equal chance of being selected in the sample.

Random Variable – A variable which can assume any one of a number of values, each of which has a fixed probability of occurrence.

Rationale – An explanation or exposition of controlling principles. The underlying reason, rational foundation.

Reliability – Reliability is a measure of the time stability of a device or products performance; whereas, quality deals with percent defective at a given time, reliability deals with failure rate-in-time for a specific item or items.

Sample – One or more units selected at random from a quantity of product to represent that quantity of product. A portion or percentage of devices from the population. A sample may be considered in statistical mathematics to be "representative" or "non-representative" of the population from which it is drawn for decision purposes.

Shock – An abrupt impact applied to a stationary object. It is usually expressed in "g" or gravity units.

Strain – To cause a change of form or size in a body by application of external force.

Strength – Ability to withstand stress or deformation; the characteristic or property of bodies by which they endure the application of force without breaking.

Stress – Any force that tends to deteriorate or degrade a component or device.

Synergism – The combined action of two or more agents such that the total effect is greater than the sum of the two effects taken independently.

Synergistic – Of or relating to, or resembling synergism.

Tractable – Easily handled, managed, or controlled.

Variable – A quantity that may assume either a number of individually distinct or separate values or any value.

Variance – The arithmetic mean of the squares of the deviations from the mean value. Sum of squares of the deviations from the typical or mean value divided by the number of events or observations.

Velocity – The time rate of change (motion) of position (e.g., inches per second, miles per hour, etc.)

Vibration – Oscillation. Any periodic motion of a body about a reference point.

APPENDIX A EXAMPLE FOR SOLUTION TO GENERAL EQUATION USING ASSUMED VALUES OF FAILURE RATES

Example: How to find n_T and C_T of the general equation.

Assume that for a certain diode when the test conditions of experiment test matrix in Table 3 are performed the following results:

$$\lambda_{P_1} = 2 \text{ percent/1000 hours (vendor "shelf-life" data)}$$

$$\lambda_{P_2} = 3.5 \text{ percent/1000 hours}$$

$$\lambda_{P_3} = 12 \text{ percent/1000 hours}$$

From Equation (33)

$$C_T = \frac{\ln \lambda_{P_3}}{\ln \lambda_{P_2} \ln 1.1117}$$

$$\ln \lambda_{P_2} = \ln 3.5 = 1.253$$

$$\ln \lambda_{P_3} = \ln 12 = 2.485$$

$$\ln 1.1117 = 0.1058 = 0.106$$

$$C_T = \frac{\ln \frac{2.485}{1.253}}{0.106} = \frac{\ln 1.9832}{0.106} = \frac{0.685}{0.106}$$

$$C_T = 6.46$$

From Equation (34)

$$n_T = \text{antilog} \left[\frac{\frac{\ln \ln \lambda_{p_3'}}{\ln \ln \lambda_{p_2'}} \ln 358 - \ln 398}{\frac{\ln \ln \lambda_{p_3'}}{\ln \ln \lambda_{p_2'}} - 1} \right]$$

$$\ln 358 = 5.880$$

$$\ln 398 = 5.986$$

$$\ln \ln \lambda_{p_2'} = \ln \ln 3.5 = \ln 1.253 = 0.225$$

$$\ln \ln \lambda_{p_3'} = \ln \ln 12 = \ln 2.485 = 0.910$$

$$n_T = \text{antilog} \left[\frac{\frac{0.910}{0.225} (5.88) - 5.986}{\frac{0.910}{0.225} - 1} \right]$$

$$n_T = \text{antilog} \frac{4.04 (5.88) - 5.986}{4.04 - 1} = \frac{23.755 - 5.986}{3.04}$$

$$n_T = \text{antilog} \frac{17.769}{3.04} = \text{antilog } 5.845$$

$$n_T = 346$$

Example: How to find n_S , C_S , and $\lambda_a e^{Ka}$.

Assume for a certain diode when all stress factors are at zero level except shock, $T_0 = 25^\circ\text{C}$, best supplier is selected when $Q = 1$ therefore, $\ln Q = 0$ and $e^{\ln Q} = 1$. When three levels of shock in 1/2:1:2 ratio is chosen and condition of experiment test matrix in Figure 3 are performed with the following results:

$$\lambda P_{S_1} = 5 \text{ failures/1000 hours when } S_1 = 1000 \text{ g}$$

$$\lambda P_{S_2} = 10 \text{ failures/1000 hours when } S_2 = 2000 \text{ g}$$

$$\lambda P_{S_3} = 250 \text{ failures/1000 hours when } S_3 = 4000 \text{ g}$$

$$Z_1 = \ln \lambda P_{S_1} = \ln 5 = 1.609$$

$$Z_2 = \ln \lambda P_{S_2} = \ln 10 = 2.303$$

$$Z_3 = \ln \lambda P_{S_3} = \ln 250 = 5.521$$

Let

$$X = \left(\frac{S_2}{n_S} \right)^{C_S}$$

$$Y = \ln \psi$$

$$\theta = 2^{C_S}$$

$$X = \frac{-(Z_1 - Z_2)(Z_3 - Z_2)}{(Z_1 - Z_2) + (Z_3 - Z_2)} = \frac{-(1.609 - 2.303)(5.521 - 2.303)}{(1.609 - 2.303) + (5.521 - 2.303)}$$

$$X = \frac{-(-0.694)(3.218)}{-0.694 + 3.218} = \frac{2.233}{2.524} = 0.884$$

$$\theta = \frac{Z_3 - Z_2 + X}{X} = \frac{5.521 - 2.303 + 0.884}{0.884} = \frac{4.102}{0.884}$$

$$\theta = 4.64$$

$$C_S = \frac{\ln \theta}{\ln 2} = \frac{\ln 4.64}{0.693} = \frac{1.535}{0.693} = 2.216$$

$$n_S = \frac{S_2}{1/C_S} = \frac{2000}{(0.884)^{1/2.216}} = \frac{2000}{0.946} = 2114$$

$$\frac{1}{2.216} \ln 0.884 = \frac{1}{2.216} (-0.123) = -0.0555$$

$$(0.884)^{1/2.216} = 0.946$$

$$\lambda P_{S_2} = \psi e^{(S_2/n_S)^{C_S}}$$

$$\psi = \frac{\lambda P_{S_2}}{e^{(S_2/n_S)^{C_S}}} = \frac{10}{e^{(2000/2114)^{2.216}}} = \frac{10}{e^{(0.946)^{2.216}}}$$

$$\psi = \frac{10}{e^{(0.884)}} = \frac{10}{2.42} = 4.132$$

We can then proceed to evaluate the value of $\lambda_a e^{K_a}$ from

$$\psi = \lambda_a e^{K_a} e^{(T_0 + 273/n_T)^{C_T}}$$

$$\lambda_a e^{Ka} = \frac{\psi}{e^{(298/346)^{6.46}}} = \frac{4.132}{e^{(0.861)^{6.46}}} = \frac{4.132}{e^{(0.382)}}$$

$$\lambda_a e^{Ka} = \frac{4.132}{1.465} = 2.813$$

We can evaluate the values of other constants n_i , C_i and constants $\lambda_a e^{Ka}$ and $\lambda_b e^{Kb}$ by the same method.

Example: How to find value of I_S

Assume that when test is performed under conditions of experiment test matrix in Table 4 is performed $\lambda_{PS8} = 800$ failures/1000 hours and previously calculated values of constants are as follows:

$$n_T = 346, C_T = 6.46, n_S = 2114, C_S = 2.216 \text{ and } \lambda_a e^{Ka} = 2.813,$$

$$T_0 = 125^\circ C$$

$$\lambda_{PS8} = \lambda_a e^{Ka} e^{(T_0 + 273/n_T)^{C_T}} e^{(S/n_S)^{C_S}} (T_0 + 273/298)^{C_S I_S}$$

$$800 = 2.813 e^{(125+273/346)^{6.46}} e^{[(2000/2114)^{2.216} (125+273/298)^{2.216 I_S}]}$$

$$800 = 2.813 e^{(398/346)^{6.46}} e^{[(0.946)^{2.216} (398/298)^{2.216 I_S}]}$$

$$800 = 2.813 e^{(1.15)^{6.46}} e^{[(0.884) (1.335)^{2.216 I_S}]}$$

$$800 = 2.813 e^{2.465} e^{[(0.884) (1.335)^{2.216 I_S}]}$$

$$800 = 2.813 (11.76) e^{[(0.884) (1.335)^{2.216 I_S}]}$$

$$800 = 33.06 e^{[(0.884)(1.335)^{2.216 I_S}]}$$

$$\frac{800}{33.06} = e^{[(0.884)(1.335)^{2.216 I_S}]}$$

$$24.2 = e^{(0.884)(1.335)^{2.216 I_S}}$$

$$\ln 24.2 = (0.884)(1.335)^{2.216 I_S}$$

$$\frac{\ln 24.2}{0.884} = (1.335)^{2.216 I_S}$$

$$\ln \left[\frac{\ln 24.2}{0.884} \right] = 2.216 I_S \ln 1.335$$

$$I_S = \frac{\ln \left(\frac{\ln 24.2}{0.884} \right)}{2.216 \ln 1.335} = \frac{\ln \left(\frac{3.186}{0.884} \right)}{2.216 (0.289)}$$

$$I_S = \frac{\ln 3.6}{0.64} = \frac{1.281}{0.64} = 2$$

We can evaluate the values of other I_i factors by the same method.

APPENDIX B

Preliminary Test Procedure -- Shocks

Procedure to determine critical plane most likely to induce failure due to shock.

When the critical plane most likely to induce failure due to shock is not known, a preliminary test should be run on a test sample of 15 diodes to establish that plane by the following method:

1. Divide the diodes into three test groups of 5 diodes each. Mount each test group on the shock-testing apparatus in 3 different planes.
2. The diodes shall be visually examined and measurements of electrical characteristics will be made prior to start of testing.
3. The diodes shall be nonoperating and shall be subjected to 4000 g's at 0.5 millisecond. One test cycle shall consist of 5 impacts or blows.
4. The diodes shall be examined for evidence of catastrophic failure such as open or short-circuiting after each test cycle. The test shall continue until all 5 diodes in a test group have failed.
5. The plane of orientation where all 5 diodes failed first will be the plane of orientation to be used to run the experiment test Matrix II, to evaluate the shock terms of the general equation.

APPENDIX C

Preliminary Test Procedure -- Vibrations

Procedure to determine critical plane most likely to induce failure due to vibration.

When the critical plane most likely to induce failure due to vibration is not known, a preliminary test should be run on a test sample of 15 diodes to establish that plane by the following method:

1. Divide the diodes into three test groups of 5 diodes each. Mount each test group on the vibration testing apparatus in 3 different planes.
2. The diodes shall be visually examined and measurements of electrical characteristics will be made prior to testing.
3. The diodes shall be nonoperating and shall be subjected to vibration amplitude of 120 g's. One test cycle shall consist of a frequency sweep between the approximate limits of 10 to 2000 cps which shall be traversed in 25 ± 5 minutes. When resonance is detected, the diodes shall be vibrated for 5 minutes at each critical resonant frequency observed. Interruptions are permitted provided the requirements for maintaining a constant peak acceleration of the specified "g" value, rate of change and test duration are met.
4. The specimens shall be examined for evidence of catastrophic failure such as open or short-circuiting after each test cycle. The testing shall continue until all 5 diodes in a test group have failed. This establishes the critical plane of orientation most likely to induce failure due to vibration.

APPENDIX D

Preliminary Test Procedure -- Acceleration

Procedure to determine critical plane most likely to induce failure due to acceleration.

When the critical plane most likely to induce failure due to acceleration is not known, a preliminary test should be run on a test sample of 15 diodes to establish that plane by the following method:

1. Divide the diodes into three test groups of 5 diodes each. Mount each test group on the acceleration-testing apparatus in 3 different planes.
2. The diodes shall be visually examined and measurements of electrical characteristics will be made prior to testing.
3. The diodes shall be nonoperating and shall be subjected to 80,000 g's acceleration level. The rate of acceleration shall be increased smoothly from zero to the specified "g" value in not less than 20 seconds. The rate of deceleration shall be decreased smoothly to zero in not less than 20 seconds. One test cycle shall consist of subjecting the diodes for 1 minute at the specified "g" level.
4. The specimens shall be examined for evidence of catastrophic failure such as open or short-circuiting after each test cycle. The testing shall continue until all diodes in a test group have failed. This establishes the critical plane of orientation most likely to induce failure due to acceleration.